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Quantitative Network Science (QNetS)

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Abstract

Quantitative network science, the science of studying and analysing complex networks, has become increasingly important and has grown significantly in recent years. Network science and network data analysis are not confined to any single scientific discipline as networks occur in diverse areas of science. Networks provide an abstract way of describing relationships and interactions between elements of complex and heterogeneous systems. For example, the World Wide Web can be represented as a network whose vertices are the HTML documents, connected by the hyperlinks that point from one page to another. On a different level, our nervous system forms a large network, whose vertices are the neurons and nerve cells, which are connected by axons. Complex networks are also considered in social and economic sciences. There the vertices represent (specific groups of) individuals or entities, and the edges describe social (or some other type of) interaction between them.

While the scientific disciplines in which networks occur are diverse, the needs for analysis are similar and may be dealt with by using the same or similar quantitative methods and models. These include methods and theories ranging from mathematical graph theory and statistical network models to visualization techniques in computer science. The CAS Research Focus „Quantitative Network Science“ intended to bundle the different activities within network science at LMU and brought together mathematicians, statisticians, and computer scientists with empirical scientists from a wide range of disciplines in order to advance the field of quantitative network science. The working group included Prof. Dr. Francesca Biagini (Mathematics), Prof. Dr. Andreas Butz (Informatics), Prof. Dr. Göran

Kauermann (Statistics), Prof. Dr. Martin Kocher (Economics), Prof. Dr. Hinrich Schütze (Linguistics), Prof. Gregory Wheeler (Philosophy), and Prof. Dr. Ralf Zimmer (Bioinformatics).

Research topics

Networks for modelling information propagation

Current estimates (www.internetworldstats.com/stats.htm) demonstrate that more than 2.8 billion people are using the Internet and its numerous services (e.g. email and the World Wide Web) on a daily basis. Especially social networks provide easily accessible ways for interaction and communication among individuals, thus making the Internet an ideal environment for the spread of all kinds of information. The dynamics of such information spreading processes constitute an important topic of study not only in mathematics, but also in other disciplines, including life sciences, computer science, economics, and sociology. Empirical observations confirm that information disseminates extremely fast, especially in the occurrence of extraordinary or unexpected events, like earthquakes, plane crashes, or other incidents originating from natural or human activities. Modern media like Facebook and Twitter are further propelling this development. The mathematical treatment of this striking phenomenon, and in particular the study of several fundamental mechanisms and the essential properties of networks that drive the process of information spreading, are a core topic.

Networks for modelling dynamic interaction

The analysis of social network data is an emerging field. The intention is to explain why two or more actors in a social or economic network interact (e.g. building

friendship or starting collaboration) and how this interaction evolves over time. A typical question is to determine what stimulates the development of an edge between nodes within a network. Though existing models for the analysis of such questions originated from sociology, the application of these models goes well beyond classical social interaction networks and also includes economic applications, such as trading or recruiting networks.

Networks for modelling flows

Economists are interested in the influence of networks on economic activities, e.g. trading flows. How do activities in networked markets evolve over time and depend on the network structure? How are relevant economic decisions influenced by the network structure? How does information among economic agents flow within networks and how does this depend on structure? What is the impact of shocks on a network of financial institutions? Many relevant research areas of economics – e.g. the evolution of cooperation, information cascades, or the intertwining of financial institutions – can be better understood when using networks to describe them. Based on theoretical predictions, field data are used to address research questions, and experiments are devised in order to understand economic and financial networks.

Networks for modelling biological interactions

In biological systems many subsystems can be represented as networks. These include, for example, metabolic, protein interaction, and cellular signal transduction networks. Cellular signal transduction networks pose a very complex and, consequently, very interesting example. Their main aim is to trigger proper response to external stimuli (including signals from other cells, but also toxic substances or pathogens). These networks are represented as dynamic networks, which frequently have highly complex cascaded structures containing many feed-forward

and feed-back loops that can amplify certain signals but suppress some others. Many diseases can be regarded as a dysfunction of these signalling pathways. For example, autoimmune disorders can be seen as a dysregulation of signalling pathways within cells of the immune system that causes stimuli coming from other cells of the same organism to be wrongly classified as a threat. This in turn provokes an immune response against the cells generating these stimuli and, consequently, the damage of these cells. Therefore dissection and understanding of cellular signalling networks is pivotal for designing novel treatments against plethora of diseases.

Networks for understanding the role of subsystems

One of the most fascinating processes in biology is the development of multicellular organisms from a single cell, the fertilized egg. In a highly orchestrated fashion, transcription factors, often in conjunction with intercellular signalling pathways, regulate genome expression in well-defined spatio-temporal patterns, which first generate the body plan of the organism and then direct the differentiation of cells and tissues. These developmental networks are studied by quantitatively determining how strongly and to which genomic regulatory sequences different transcription factors are binding and how they influence the expression of their target genes. One important question is to understand how the biological system is dynamically driven through many different intermediary states until stable states of differentiation are reached.

Networks for modelling connection structures

In computational linguistics, systems that automatically process natural language make use of data stores that contain knowledge about language and the world. Such data stores are called linguistic or knowledge resources. Many such resources are networks. One example is *WordNet*, a computational lexicon of English. The two most important node types in *WordNet* are word forms

and so-called synsets (roughly: meanings). For example, “suit” is a word form that is part of two synsets, one referring to clothes, one referring to lawsuits. In the *WordNet* network, the word form “suit” is connected with the meanings “clothes” and “lawsuits”. Since further progress in language processing depends on improved resources, the analysis of resources that take the shape of networks is of central importance in computational linguistics.

Networks in philosophy

Philosophy has a long tradition of questioning concepts and methods that are taken for granted, as well as of challenging answers to questions that appear long settled. For example, in Plato’s *Republic*, Socrates grapples with the question of why it is rational to behave justly or how we come to agree on what words refer to without already having a language, and he does this in the manner we have come to expect from a Socratic dialogue: by engaging with a stooge. For the rationality of justice and agreement of language meaning, each question concerns multiple individuals who interact with one another and whose well being depends on everyone else’s actions within the group, and how well they coordinate actions. Recently, philosophers have revisited the questions of emergent norms and signalling systems in terms of agent based models that facilitate modelling the dynamics of heterogeneous, boundedly rational agents interacting locally within a network.

Fellows and Activities

The CAS Research Focus QNetS comprised several activities designed to foster interdisciplinary cooperation and cross-fertilisation in the field of quantitative network science both at LMU Munich and beyond.

(a) Visiting Fellows

In order to provide new interdisciplinary perspectives within the field of network science, both senior and

junior researchers from different disciplines were invited and thus new cooperation were promoted.

Prof. Marco Maggis, Ph.D. (November – December 2015), assistant professor at the Department of Mathematics at the Università degli Studi di Milano is a researcher who focuses on functional/convex analysis and stochastic calculus applied to mathematical finance. His work also addresses the problem of risk quantification in the presence of uncertainty.

Prof. Dr. Ulrik Brandes (February – April 2016) is Professor for Social Networks in the Department of Humanities, Social and Political Sciences at ETH Zurich. He is a leading expert in network analysis and visualization, with extensive interdisciplinary collaboration and a particular interest in social networks.

Prof. Yan Chen, Ph.D. (July – August 2016) is Daniel Kahneman Collegiate Professor of Information at the University of Michigan. Her research interests are behavioural and experimental economics, mechanism design, and information economics. The fundamental challenge her research addresses is the design of robust economic mechanism when agents are not perfectly rational. She is an expert in the experimental investigation of behaviour within network, of matching of agents, and of social norms. Furthermore, she has been interested in economic behaviour within social networks outside the laboratory.

(b) Lectures

By offering a series of lectures, accessible to a wider academic public, the CAS Research Focus QNetS tried to raise public awareness of this rather complex matter, which – though technically difficult to be understood for non-experts – has a dramatic impact on everybody’s life.

■ Model Risk, Solvency and Risk Aggregation (November 2015)

Prof. Dr. Paul Embrechts (Mathematics, ETH Zurich); Chair: Prof. Dr. Francesca Biagini (Financial Mathematics, LMU Munich).

■ How Do Neuronal Circuits Operate? (January 2016)

Prof. Dr. Alexander Borst (Circuits – Computation – Models, MPI of Neurobiology), Dr. Moritz Helmstädter (Connectomics, MPI for Brain Research); Chair: Prof. Dr. Martin Wirsing (Informatics, LMU Munich).

■ Managing Systematic Risk in Financial Multilayer Networks (October 2016)

Prof. Dr. Stefan Thurner (Science of Complex Systems, Medical University of Vienna); Chair: Prof. Dr. Paul Thurner (Political Sciences, LMU Munich).

■ Using Computational Tools to Uncover Structure and Status in Academic Fields and Public Debats. (April 2016)

Dr. Achim Edelmann (Sociology, Bern University), Prof. Kieran Healy, Ph.D. (Ethics, Duke University); Chair: Prof. Dr. Göran Kauermann (Statistics and its Applications in Economics and Social Sciences, LMU Munich).

(c) Workshops

The CAS Research Focus QNetS organized two workshops, which strengthened the already existing cooperation between the different disciplines involved in the CAS Research Focus at LMU Munich and fostered interdisciplinary cooperation and cross-fertilisation on an international level.

■ Computational Methods for Networks (November 2015)

Quantitative network science, the science of analysing networks, has become increasingly important in the past years. It is not confined to any single scientific discipline but similar quantitative needs arise in various fields such as, for example, statistics, mathematics, informatics, physics, engineering, genetics, medicine, biology, and social sciences. This workshop provided an interdisciplinary overview on how network analysis is currently used to address different research questions in various scientific areas at LMU Munich. **Invited speakers were** Ziga Avsec (LMU), Ulrik Brandes (ETH

Zurich), Nils Detering (LMU), Claudia Klüppelberg (TUM), Christian Ochsenfeld (LMU), Sascha Rothe (LMU), Thomas Seidl (LMU), Paul Thurner (LMU), Mike West (Duke University/TUM), and Hendrik Zipse (LMU).

■ What's New in Networks? – Building Bridges between Computational, Mathematical and Statistical Network Analysis (October 2016)

The workshop provided an overview of the current research in network analysis in the fields of mathematics, informatics, and statistics, with applications in medicine, bioinformatics, and computational biology. The workshop brought together international researchers from various academic disciplines in order to foster cooperation and cross-fertilisation between these these different fields. **Invited speakers were** Steffen Dereich (University of Münster), Aristides Gionis (Aalto University), Remco van der Hofstad (TU Eindhoven), David Hunter (Penn State), Trey Ideker (UC San Diego), Dmitri Krioukov (Northeastern University), Michael Sedlmair (University of Vienna), Ron Shamir (Tel Aviv University), and Alfonso Valencia (Madrid).

Summary

The CAS Research Focus gathered researchers in network analysis from a variety of disciplines at LMU Munich and thus established a platform for communication and future collaborations. Through the various activities within the CAS Research focus already existing interdisciplinary research of quantitative sciences at LMU Munich was fostered. Additionally co-operations with scientists at LMU Munich, as well as in Germany and abroad were initiated. The research focus allowed us to assess the relevance of the subject matter in current research both on a theoretical and on an applied level. We could demonstrate how network analysis can provide a common language for completely different academic disciplines. Finally, the research focus contributed to the international visibility of LMU Munich in the field of quantitative network analysis.

Further reading

- Amini, H, Cont, R, Minca, A (2016). Resilience to contagion in financial networks. *Mathematical Finance*, 26 (2), 329-365.
- Amini, H, Minca, A (2016). Inhomogeneous Financial Networks and Contagious Links. *Operations Research*, 5 (64), 1109-1120.
- Baccini, L, Lenzi, V, Thurner, PW (2013). Global Energy Governance: Trade, Infrastructure, and the Diffusion of International Organizations. *International Interactions*, 39 (2), 192-216.
- Biagini, F, Fouque, J-P, Frittelli, M, Meyer-Brandis, T (2017). A unified approach to systemic risk measures via acceptance sets. *Mathematical Finance*. (accepted)
- Biagini, F, Kauermann, G (eds.) (in prep.) *What's new in Network Science – An Aerial View from Different Perspectives*. Springer.
- Biagini, F, Mazzon, A, Meyer-Brandis, T (2016). Liquidity induced asset bubbles via flows of ELMs. *Mathematical Finance*. (preprint)
- Boergens, KM, Berning, M, Bocklisch, T, Bräunlein, D, Drawitsch, F, Frohnhofen, J, Herold, T, Otto, P, Rzepka, N, Werkmeister, DW, Wiese, G, Wissler, H, Helmstaedter, M (2017). WebKnossos: Efficient Online 3D Data Annotation for Connectomics. *Nature Methods*, 14, 691-694.
- Bookmann, B, Thurner, PW (2006). Flexibility Provisions in Multilateral Environmental Treaties. *International Environmental Agreements. Politics, Law and Economics*, 6 (2), 113-135.
- Borst, A, Helmstaedter, M (2015). Common circuit design in fly and mammalian motion vision. *Nature Neurosci*, 18, 1067-1076.
- Brandes, U (2016). Network Positions. *Methodological Innovations*, 9, 1-19.
- Capponi, A, Chen, P-C, Yao, DD (2015). Liability Concentration and Systemic Losses in Financial Networks. *Operations Research*, 64 (5), 1121-1134.
- Charness, G, Feri, F, Meléndez-Jiménez, MA, Sutter, M (2014). Experimental Games on Networks: Underpinnings of Behavior and Equilibrium Selection. *Econometrica*, 82, 1615-1670.
- Cont, R, Moussa, A, Santos, EB (2013). Network structure and systemic risk in banking systems. In: Fouque, J-P, Langsam, J (eds.), *Handbook on Systemic Risk*, Cambridge: Cambridge University Press, 327-368.
- Csereklyei, Z, Thurner, PW, Bauer, A, Küchenhoff, H (2016). The Effect of Economic Growth, Oil Prices, and the Benefits of Reactor Standardization: Duration of Nuclear Power Plant Construction Revisited. *Energy Policy*, 91, 49-59.
- Detering, N, Meyer-Brandis, T, Panagiotou, K (2016). Bootstrap percolation in directed and inhomogeneous random graphs. (preprint)
- Detering, N, Meyer-Brandis, T, Panagiotou, K, Ritter, D (2017). Managing default contagion in inhomogeneous financial networks. (preprint)
- Durrani, N, Schmid, H, Fraser, AM, Koehn, P, Schütze, H (2015). The Operation Sequence Model – Combining N-Gram-Based and Phrased-Based Statistical Machine Translation. *Computational Linguistics*, 41 (2), 185-214.
- Eisenberg, L, Noe, TH (2001). Systemic Risk in Financial Systems. *Management Science*, 47 (2), 236-249.
- Gai, P, Kapadia, S (2010). Contagion in Financial Networks. *Proceedings of the Royal Society A*, 466, 2401-2423.
- Gleeson, JP, Hurd, TR, Melink, S, Heckett, A (2013). *Advances in Network Analysis and its Applications. Systemic risk in banking networks without monte carlo simulation*. Springer.
- Helmstaedter, M (2015). The mutual inspirations of machine learning and neuroscience. *Neuron*, 86(1), 25-28.
- Helmstaedter, M (2013). Cellular-resolution connectomics: challenges of dense neural circuit reconstruction. *Nature Methods*, 10, 501-507.

- Hoffmann, H, Meyer-Brandis, T, Svindland, G (2017). Allocation of systemic risk. (preprint)
- Hoffmann, H, Meyer-Brandis, T, Svindland, G (2016). Risk-consistent conditional systemic risk measures. *Stochastic Processes and their Applications*, 126 (7), 2014-2037.
- Hurd, TR (2016). *Contagion! Systemic Risk in Financial Networks*. Heidelberg: Springer.
- Jackson, MO (2010). *Social and Economic Networks*. Princeton: Princeton University Press.
- Joesch, M, Schnell, B, Raghu, SV, Reiff, DF, Borst, A (2010). ON and OFF pathways in Drosophila motion vision. *Nature*, 468, 300-304.
- Klima, A, Küchenhoff, H, Selzer, M, Thurner, PW (2017). *Exit Polls und Hybrid-Modelle. Ein neuer Ansatz zur Modellierung von Wählerwanderungen*. Springer.
- Klima, A, Schlesinger, T, Thurner, PW, Küchenhoff, H (2017). Combining Aggregate Data and Individual Exit Polls for the Estimation of Voter Transitions. *Sociological Methods and Research*, 1-30.
- Klima, A, Thurner, PW, Molnar, C, Schlesinger, T, Küchenhoff, H (2016). Estimation of Voter Transitions Based on Ecological Inference: An Empirical Assessment of Different Approaches. *Advances in Statistical Analysis*, 100 (2), 133-159.
- Maisak, MS, Haag, J, Ammer, G, Serbe, E, Meier, M, Leanhardt, A, Schilling, T, Bahl, A, Rubin, GM, Nern, A, Dickson, BJ, Reiff, DF, Hopp, E, Borst, A (2013). A directional tuning map of Drosophila elementary motion detectors. *Nature*, 500, 212-216.
- Maurer, I, Pöbnecker, W, Thurner, PW, Tutz, G (2014). Modeling electoral choices in multiparty systems with high-dimensional data: A regularized selection of parameters using the Lasso approach. *Journal of Choice Modelling*, 16, 23-42.
- Maurer, I, Pöbnecker, W, Thurner, PW, Tutz, G (2015). Modeling electoral choices in multiparty systems with high-dimensional data: A regularized selection of parameters using the Lasso approach. *Journal of Choice Modelling*, 16, 23-42.
- Maurer, I, Thurner, PW, Debus, M (2015). Under Which Conditions do Parties Attract Voters' Issue Reactions? Party Varying Issue Voting in German Election 1987-2009. *West European Politics*, 28 (6), 1251-1273.
- Mauss, AS, Pankova, K, Arenz, A, Nern, A, Rubin, GM, Borst, A (2015). Neural circuit to integrate opposing motions in the visual field. *Cell*, 162, 351-362.
- Mueller, WC, Thurner, PW (2017). *Politics of Nuclear Energy in Western Europe*. Oxford University Press: Oxford.
- Pamp, O, Thurner, PW (2017). Trading Arms and the Demand for Military Expenditures: Empirical Explorations Using New SIPRI-Data. *Defence and Peace Economics*, 28, 457-472.
- Pappi, FU, Thurner, PW (2000). Die deutschen Wähler und der Euro: Auswirkungen auf die Bundestagswahl 1998 und längerfristige Perspektiven. *Politische Vierteljahresschrift*, 41 (3), 435-465.
- Pappi, FU, Thurner, PW (2002). Electoral Behavior in a Two-Vote- System: Incentives for Ticket Splitting in German Bundestag Elections. *European Journal of Political Research*, 41 (2), 207-232.
- Poledna, S, Bochmann, O, Thurner, S (2017). Basel III capital surcharges for G-SIBs are far less effective in managing systemic risk in comparison to network-based, systemic risk-dependent financial transaction taxes. *Journal of Economic Dynamics and Control*, 77, 230-246.
- Poledna, S, Molina-Borboa, JL, Martínez-Jaramillo, S, van der Leij, M, Thurner, S (2015). Multi-layer network nature of systemic risk in financial networks and its implications. *Journal of Financial Stability*, 20, 70-81.
- Poledna, S, Thurner, S (2016). Elimination of systemic risk in financial networks by means of a systemic risk transaction tax. *Quantitative Finance*, 16 (10), 1599-1613.

- Rothe, S, Schütze, H (2015). AutoExtend: Extending Word Embeddings to Embeddings for Synsets and Lexemes. *Proceedings of the ACL*, 1, 1793-1803.
- Sajjad, H, Schmid, H, Fraser, A, Schütze, H (2017). Statistical Models for Unsupervised, Semi-Supervised, and Supervised Transliteration Mining. *Computational Linguistics*, 43 (2), 349-375.
- Schosch, D, Brandes, U (2016). Re-Conceptualizing Centrality in Social Networks. *European Journal of Applied Mathematics*, 27 (6), 971-985.
- Schmidt, H, Gour, A, Straehle, J, Boergens, KM, Brecht, M, Helmstaedter, M (2017). Axonal synapse sorting in medial entorhinal cortex. *Nature*, 549, 469-475.
- Schütze, H, Yaghoobzadeh, Y (2017). Multi-level Representations for Fine-Grained Typing of Knowledge Base Entities. *Proceedings of the 15th Conference of the EACL, Vol. 1*, 578-589.
- Shikano, S, Herrmann, M, Thurner, PW (2009). Strategic Voting under Proportional Representation: Threshold Insurance in German Elections. *West European Politics*, 32 (3), 634-656.
- Thiemichen, S, Friel, N, Alberto, C, Kauermann, G (2016). Bayesian Exponential Random Graph Models with Nodal Random Effects. *Social Networks*, 46, 11-28.
- Thiemichen, S, Kauermann, G (2017). Stable Exponential Random Graph Models with Non-parametric Components for Large Dense Networks. *Social Networks*, 49, 67-80.
- Thurner, PW (2000). The Empirical Application of the Spatial Theory of Voting in Multiparty Systems with Random Utility Models. *Electoral Studies*, 19 (4), 493-517.
- Thurner, PW (2005). Review of: Thomas Bräuninger, Internationale Institutionenpolitik: Die Wahl von Entscheidungsregeln für die Meeresbodenbehörde. Frankfurt/New York: Campus, 2002. *Kölner Zeitschrift für Soziologie und Sozialpsychologie*, 57 (2), 356-358.
- Thurner, PW (2008). Nichtförmliche Verwaltungskooperation der Ministerialbürokratien der EU-Mitgliedstaaten und netzwerkbasierte Zusammenstellung von nationalen und übernationalen Prozessteams. *Politische Vierteljahresschrift*, 41, 401-429.
- Thurner, PW (2008). Review of: Hofmann, HCH, Türk, AH (eds.), EU Administrative Governance, 2006. *European Journal of International Law*, 19 (2), 455-456.
- Thurner, PW (2017). Networks in European Union Politics. In: Victor, JN, Alexander H. Montgomery, AH, Lubell, M (eds.): *Oxford Handbook on Political Networks*. Oxford: Oxford University Press.
- Thurner, PW, Binder, M (2009). EU Trans-governmental Networks: The Emergence of a New Political Space beyond the Nation State? *European Journal of Political Research*, 48 (1), 80-106.
- Thurner, PW, Binder, M (2012). Formal and Real Authority of Ministerial Jurisdictions in Semi-Open States. The Comparative Value of Inside and Outside Networking. *Journal of European Public Policy*, 19 (6), 817-843.
- Thurner, PW, Eymann, A (2000). Policy-Specific Alienation and Indifference in the Calculus of Voting: A Simultaneous Model of Party Choice and Abstention. *Public Choice*, 102 (1-2), 51-77.
- Thurner, PW, Klima, A, Küchenhoff, H (2015). Agricultural Structure and the Rise of the Nazi Party Reconsidered. *Political Geography*, 44, 50-63.
- Thurner, PW, Linhart, E (2004). Political Multilevel Negotiations and Issue Linkage during an EU Intergovernmental Conference: An Empirical Application. *Computational and Mathematical Organization Theory*, 10 (3), 243-266.
- Thurner, PW, Mittermeier, L, Küchenhoff, H (2014). How Long does it Take to Build a Nuclear Power Plant? A Non-parametric Event History Approach with P-splines. *Energy Policy*, 70, 163-171.
- Thurner, PW, Pappi, FU (2006). Domestic and International Politics during EU Intergovernmental

Conferences. Bridging the Gap between Negotiation Theory and Practice. *Negotiation Journal*, 22 (2), 167-185.

- Thurner, PW, Pappi, FU (2013). *European Union Intergovernmental Conferences: Domestic Preference Formation, Transgovernmental Networks, and the Dynamics of Compromise*. New York/ London: Routledge.
- Thurner, PW, Stoiber, M (2002). Interministerielle Netzwerke: Formale und informelle Koordinationsstrukturen bei der Vorbereitung der deutschen Verhandlungspositionen zur Regierungskonferenz 1996. *Politische Vierteljahresschrift*, 42 (2), 561-605.
- Thurner, PW, Stoiber, M, Weinmann, C (2005). Informelle Transgouvernementale Koordinationsnetzwerke der Ministerialbürokratie der EU-Mitgliedstaaten bei einer Regierungskonferenz. *Politische Vierteljahresschrift*, 43 (4), 552-574.
- Thurner, S, Poledna, S (2013). DebtRank-transparency: Controlling systemic risk in financial networks. *Scientific Reports*, 3. doi:10.1038/srep01888
- Yin, W, Schütze, H, Xiang, B, Zhou, B (2016). ABCNN: Attention-Based Convolutional Neural Network for Modeling Sentence Pairs. *TACL*, 4, 259-272.